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Third generation Yukawa couplings unification in supersymmetric SO(10) model

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Abstract. In the minimal supersymmetric standard model (MSSM) contained in SUSY SO(10), top-b-τ Yukawa unification is achieved at the intermediate mass scale $M_I \simeq 10^{13.10}$ GeV using the recent world average experimental value of the top-quark mass, $m_{\text{top}} = 175 \pm 6$ GeV, which has been directly established by CDF and D0 experiments at the Tevatron Collider. It is also observed that the Yukawa couplings unification scale M_I can be further decreased by taking lower input values of the top-quark mass. This trend indicates the possible existence of an intermediate symmetry breaking scale in SUSY SO(10). The present finding does not agree with the earlier notion that the third generation Yukawa couplings unification should occur at the GUT scale M_U.

1 Introduction

The topic of Yukawa couplings unification in supersymmetric unified theory (GUT) framework motivated by SO(10) or E_6 unification, has been given considerable attention over the last few years [1]. Minimal supersymmetric standard model (MSSM) has been accepted as a promising theory at least on its three crucial predictions consistent with the latest experimental measurements: (i) the value of $\text{Sin}^2\theta_w(M_Z)$, (ii) the meeting of the three gauge coupling constants when extrapolated at higher energy scale, and (iii) the high observed value of GUT scale $M_U \simeq 2 \times 10^{16}$ GeV sufficient to prevent fast proton decay in agreement with the present experimental bound [2]. The hypothesis that the third generation Yukawa couplings should meet at GUT scale ($h_{\text{top}} = h_b = h_\tau$) is in fact related to the problem of the origin of fermion masses [3]. In all earlier works, this condition is used as a starting point to predict either the top or bottom quark mass [1, 3–5]. Another important aspect of SUSY SO(10) is its symmetry breaking pattern. In the conventional SUSY SO(10) employing the Higgs supermultiplets *54*, $16_H + \overline{16}_H$ and *10* in the usual fashion, it is impossible to achieve the intermediate scale M_I substantially lower than the unification scale. Lee and Mohapatra [7] have advocated the possibility of the existence of an intermediate symmetry breaking scale around $M_I \simeq 10^{11}$–10^{12} GeV corresponding to $B-L$ symmetry breaking in SUSY GUT such as SUSY SO(10) in order to solve the strong CP problem through Peccei-Quinn mechanism and achieve small neutrino masses necessary to understand solar neutrino flux and/or the dark matter of the universe. This corresponds to the intermediate symmetry group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ ($= G_{2213}$) [7, 8]. It has also been noted [6] that the intermediate symmetry $SU(2)_L \times SU(2)_R \times SU(4)_C$ ($= G_{224P}$) can also be achieved at $M_I \simeq 5 \times 10^{12}$-$2 \times 10^{14}$ GeV, provided certain states in the adjoint representation *45* and/or $16_H + \overline{16}_H$ have masses near 1 TeV.

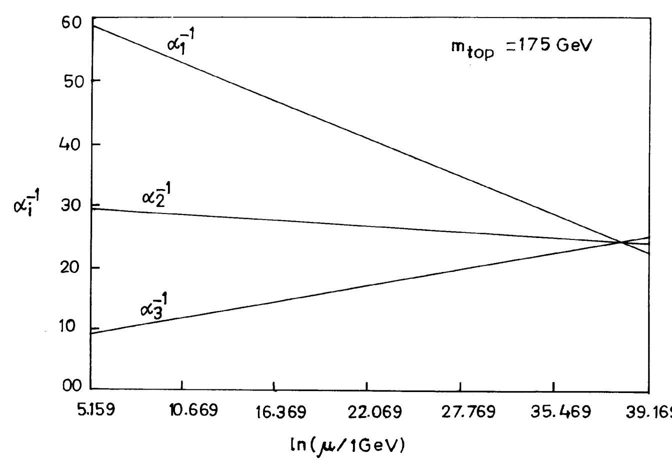
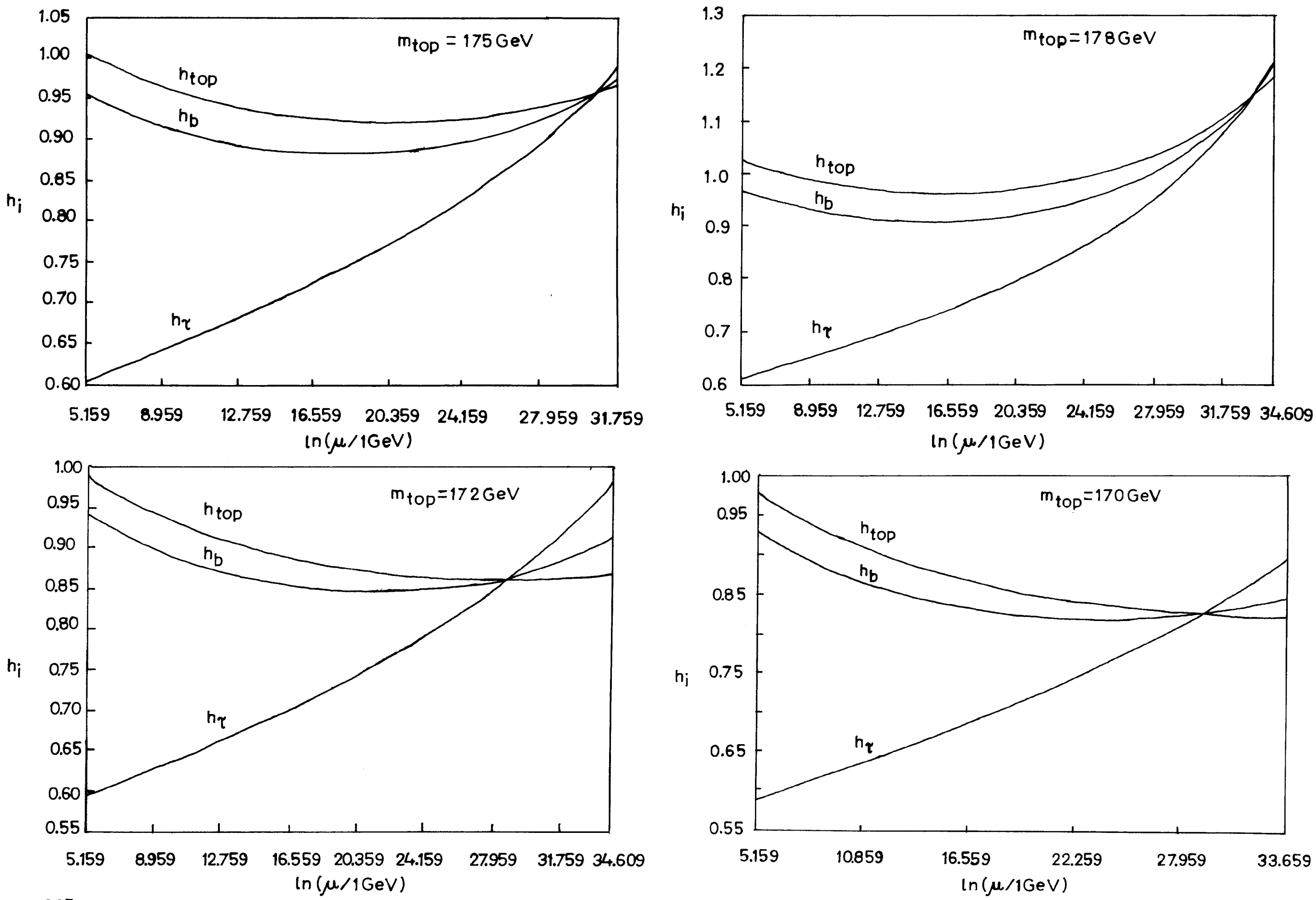


Fig. 1. Running of the three gauge couplings of the minimal SUSY standard model with mass scale $t = \ln(\mu/1\,\text{GeV})$ for $m_{\text{top}} = 175$ GeV

In the context of the precise measurements [9] of the top-quark mass by CDF and D0 experiments at the Tevatron Collider, it is now possible to study the existence of such intermediate symmetry breaking scale through the Yukawa couplings unification scheme in SUSY SO(10), and settle the question whether top-b-τ Yukawa couplings unification can occur at the intermediate scale other than GUT scale, by solving the renormalisation group equations (RGEs) of the Yukawa couplings evolving from lower scale to unification scale.



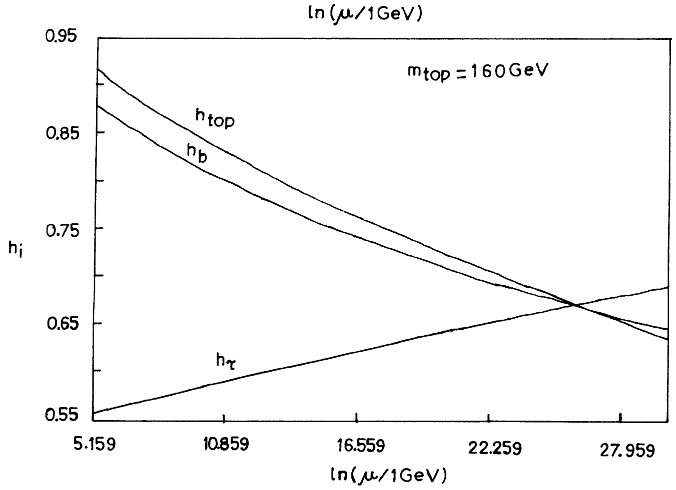


Fig. 2. Variation of the third generation Yukawa couplings as a function of mass scale $t = \ln(\mu/1\,\mathrm{GeV})$ and $\tan\beta$, exhibiting quark-lepton unification, for **a** $m_{\mathrm{top}} = 175\,\mathrm{GeV}$, $\tan\beta \simeq 59.68$; **b** $m_{\mathrm{top}} = 178\,\mathrm{GeV}$, $\tan\beta \simeq 60.43$; **c** $m_{\mathrm{top}} = 172\,\mathrm{GeV}$, $\tan\beta \simeq 58.96$; **d** $m_{\mathrm{top}} = 170\,\mathrm{GeV}$, $\tan\beta \simeq 58.15$; **e** $m_{\mathrm{top}} = 160\,\mathrm{GeV}$, $\tan\beta \simeq 55.13$

2 Formalism

The purpose of this brief report is to examine whether the idea of the third generation Yukawa couplings unification in MSSM is realisable at mass scales significantly lower than those investigated earlier [1, 3] but consistent with the recent experimental determination of the input parameters [10] and the latest value of the top-quark mass which has been measured directly at the Tevatron [9]

$$\mathrm{Sin}^2\theta_w(M_Z) = 0.2310 \pm 0.0003$$
$$\alpha_3(M_Z) = 0.118 \pm 0.004$$
$$\alpha^{-1}(M_Z) = 127.9 \pm 0.1$$
$$m_{\mathrm{top}} = 175 \pm 6\,\mathrm{GeV}$$
$$m_b = 4.25\,\mathrm{GeV}$$
$$m_\tau = 1.785\,\mathrm{GeV}. \qquad (1)$$

We use the one-loop renormalisation group equations (RGEs) for the top-quark, bottom-quark and the τ-lepton Yukawa couplings in the MSSM in the range of mass scales,

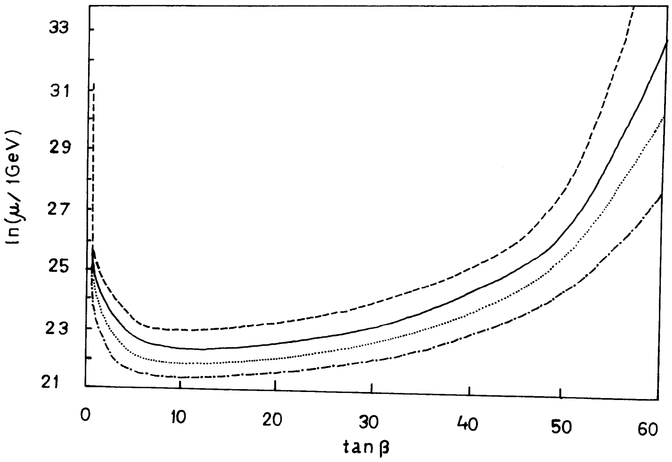


Fig. 3. Variation of the quark-lepton unification scale as a function of $\tan\beta$ for different m_{top} values: 160 GeV (*dotted-dashed line*), 170 GeV (*dotted line*), 175 GeV (*solid line*) and 180 GeV (*dashed line*)

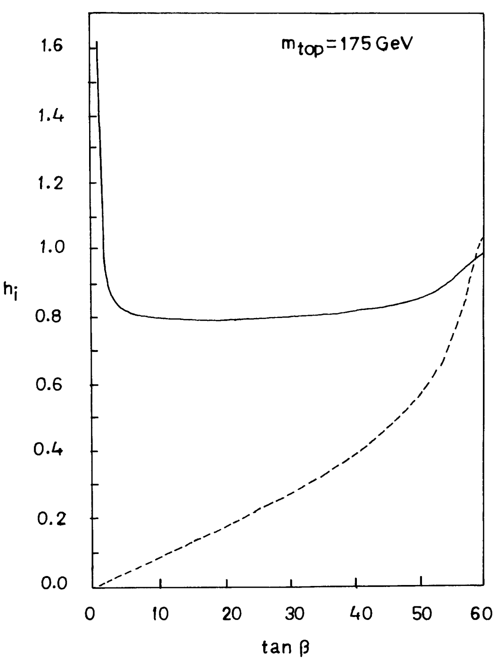


Fig. 4. Variation of $h_b = h_\tau$ unification (*dotted line*) and h_{top} (*solid line*) as a function of $\tan\beta$ for $m_{\text{top}} = 175$ GeV

$m_{\text{top}} \le \mu \le M_I$ [3, 4]

$$\begin{aligned} dh_{\text{top}}/dt &= h_{\text{top}}[6h_{\text{top}}^2 + h_b^2 - \Sigma C_i g_i^2]/(16\pi^2), \\ dh_b/dt &= h_b[6h_b^2 + h_\tau^2 + h_{\text{top}}^2 - \Sigma C_i' g_i^2]/(16\pi^2), \\ dh_\tau/dt &= h_\tau[4h_\tau^2 + 3h_b^2 - \Sigma C_i'' g_i^2]/(16\pi^2) \end{aligned} \tag{2}$$

where $i = Y, 2L, 3C$, $t = \ln\mu$, h_j = Yukawa coupling of the j^{th} fermion, and

$$\begin{aligned} C_i &= [13/15, \ 3, \ 16/3] \\ C_i' &= [7/15, \ 3, \ 16/3] \\ C_i'' &= [9/5, \ 3, \ 0] \end{aligned} \tag{3}$$

The RGEs for the gauge couplings are similarly expressed as [3, 4]

$$dg_i(t)/dt = g_i^3 \left[b_i + \sum_{j=1}^{3} b_{ij} g_j^2 - \sum_{j=\text{top},b,\tau} a_{ij} h_j^2 \right] /(16\pi^2) \tag{4}$$

where

$$\begin{aligned} b_i &= [33/5, \ 1, \ -30] \\ b_{ij} &= \begin{bmatrix} 7.96 & 5.4 & 17 \\ 1.8 & 25 & 24 \\ 2.2 & 9 & 14 \end{bmatrix} \\ a_{ij} &= \begin{bmatrix} 26/5 & 14/5 & 18/5 \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{bmatrix} \end{aligned} \tag{5}$$

Using mass-operator renormalisation of $SU(3)_C \times U(1)_{\text{em}}$ theory below M_Z, the QCD-QED rescaling factors for $m_b(m_{\text{top}})$ and $m_\tau(m_{\text{top}})$ are defined as [4]

$$\eta_b = 1.53, \quad \eta_\tau = 1.015 \tag{6}$$

where $m_i(m_{\text{top}}) = m_i(m_i)/\eta_i$, $i = b, \tau$. Following the standard procedure we obtain the top, bottom and the τ-Yukawa couplings at the top-quark mass scale $\mu = m_{\text{top}}$, $(t_0 = \ln m_{\text{top}})$

$$\begin{aligned} h_{\text{top}}(t_0) &= m_{\text{top}}(t_0)/(174\,\text{Sin}\beta), \\ h_b(t_0) &= m_b(t_0)/(174\,\eta_b \text{Cos}\beta), \\ h_\tau(t_0) &= m_\tau(t_0)/(174\,\eta_\tau\,\text{Cos}\beta) \end{aligned} \tag{7}$$

which, for a given input value of $\tan\beta$, fix the initial values of the Yukawa couplings at the top quark mass scale as input needed for numerical solution of RGEs, (2) and (4). Starting with the gauge coupling constants corresponding to the CERN-LEP measurements given in (1),

$$\begin{aligned} \alpha_1^{-1}(M_Z) &= 58.967 \pm 0.1 \\ \alpha_2^{-1}(M_Z) &= 29.622 \pm 0.06 \\ \alpha_3^{-1}(M_Z) &= 8.475^{+0.30}_{-0.29} \end{aligned} \tag{8}$$

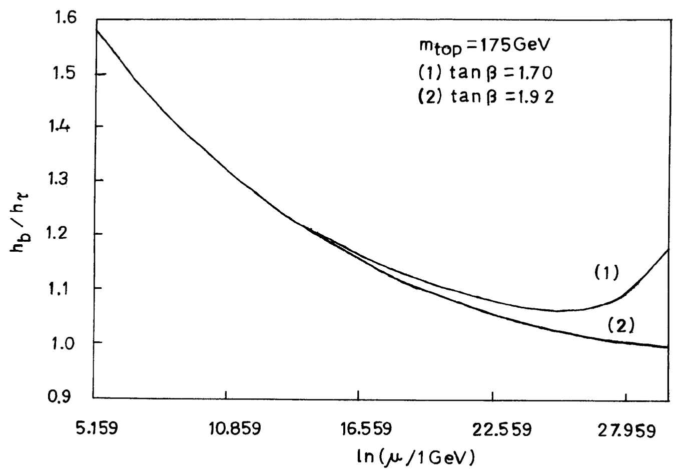
we evaluate three gauge coupling constants at $\mu = m_{\text{top}}$ in the one-loop approximation assuming the existence of one-light Higgs doublet ($n_H = 1$) and five quark flavours ($n_f = 5$) with the one-loop β-function coefficients for the three gauge couplings

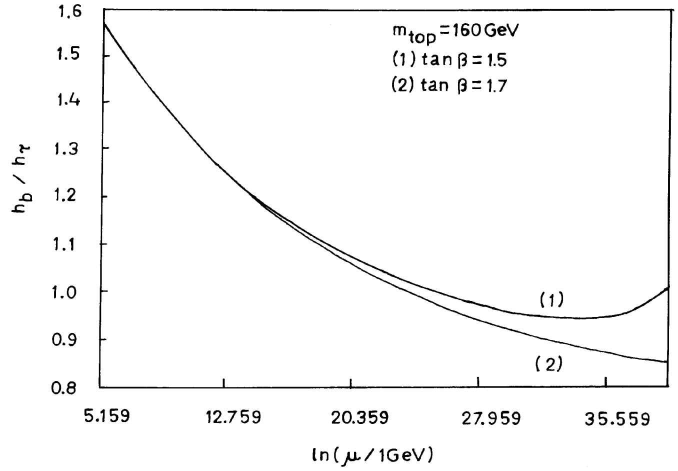
$$b_i = [53/10, \ -1/2, \ -4]$$

in the SUSY standard model [2]. This gives rise to

$$\begin{aligned} \alpha_1^{-1}(m_{\text{top}}) &= 58.417 \pm 0.1 \\ \alpha_2^{-1}(m_{\text{top}}) &= 29.673 \pm 0.06 \\ \alpha_3^{-1}(m_{\text{top}}) &= 8.889^{+0.30}_{-0.29}. \end{aligned} \tag{9}$$

These are taken to be the initial values of gauge couplings for numerical solutions to the RGEs. We follow the standard Runge-Kutta method for the simultaneous numerical solutions to the RGEs for Yukawa couplings, (2) and the gauge couplings, (4).





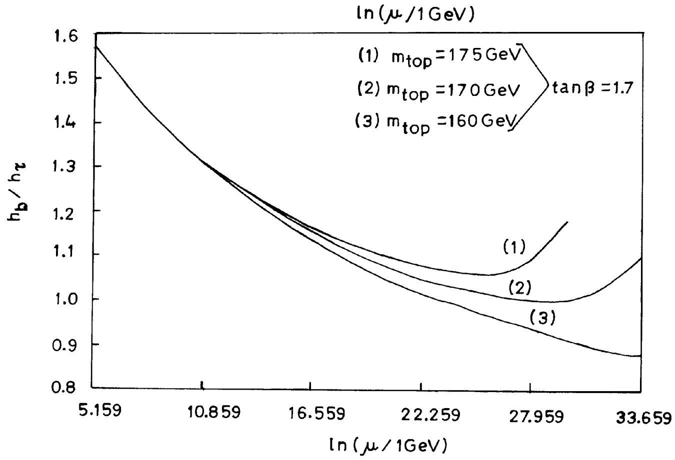


Fig. 5. Variation of the ratio h_b/h_τ as a function of mass scale $t = \ln(\mu/1\,\mathrm{GeV})$ for **a** smaller values of $\tan\beta = 1.70$, 1.92 with $m_{\rm top} = 175\,\mathrm{GeV}$, **b** $\tan\beta = 1.5$, 1.7 with $m_{\rm top} = 160\,\mathrm{GeV}$, and **c** various values of $m_{\rm top} = 175\,\mathrm{GeV}$, 170 GeV, 160 GeV with $\tan\beta = 1.7$

3 Results and discussion

As a test of our numerical solution we have verified that extrapolation of the gauge couplings including two-loop contributions through (4), exhibits unification at $t_x = t_U \simeq 37.5$ which corresponds to $M_U \simeq 2 \times 10^{16}\,\mathrm{GeV}$, (Fig. 1). The values of gauge and Yukawa couplings are thus obtained at different input values of $\tan\beta$. We search the parameter space in the larger $\tan\beta$ region taking the precise Tevatron value of the top quark masses $m_{\rm top} = 175\,\mathrm{GeV}$ and the top-b-τ Yukawa coupling unification scale is observed at $M_I \simeq 10^{13.10}\,\mathrm{GeV}$ for $\tan\beta \simeq 59.68$ as shown in Fig. 2a.

The Yukawa couplings unification scale M_I is found to be very sensitive to the input value for the top-quark mass. As a demonstration we also investigate the variation of the scale M_I with input values of $m_{\rm top} = 178$, 172, 170 and 160 GeV where the first two are the individual central values of the CDF and D0 experiments [9]. Our results are shown in Figs. 2b,c,d,e where unification of the three Yukawa couplings is clearly exhibited for the four cases: (i) $m_{\rm top} = 178\,\mathrm{GeV}$ with $\tan\beta \simeq 60.43$ at $M_I \simeq 10^{14.45}\,\mathrm{GeV}$, (ii) $m_{\rm top} = 172\,\mathrm{GeV}$ with $\tan\beta \simeq 59.96$ at $M_I \simeq 10^{12.39}$ GeV; and (iii) $m_{\rm top} = 170\,\mathrm{GeV}$ with $\tan\beta \simeq 58.15$ at $M_I \simeq 10^{12.15}\,\mathrm{GeV}$, and (iv) $m_{\rm top} \simeq 160\,\mathrm{GeV}$ with $\tan\beta \simeq 55.13$ at $M_I \simeq 10^{11.56}$ GeV respectively. The Yukawa unification scale decreases with the decrease of the input value of the top-quark mass.

In Fig. 3 we have shown the variation of the Yukawa unification scale M_I as a function of $\tan\beta$ where in the larger(smaller) $\tan\beta$ region, the values of M_I and $\tan\beta$ correspond to the unification of top-b-τ (b-τ) Yukawa couplings. The dashed-dotted, dotted, solid and dashed lines correspond to $m_{\rm top} = 160\,\mathrm{GeV}$, 170 GeV, 175 GeV and 180 GeV, respectively. From the larger $\tan\beta$ region of Fig. 3, it is clear that, for a fixed $\tan\beta$, the Yukawa unification scale increases with $m_{\rm top}$; for a fixed $\tan\beta$, the Yukawa unification scale increases with $\tan\beta$, although the rate of increase is different in different regions of curve.

In Fig. 4 we have shown the variation of $h_b = h_\tau$ (dotted line) and $h_{\rm top}$ (solid line) as a function of $\tan\beta$ for $m_{\rm top} = 175$ GeV. Unification $h_b = h_\tau$ occurs in the wide range of $\tan\beta$, but $h_{\rm top} = h_b = h_\tau$ occurs at around $\tan\beta \simeq 59.68$ for $m_{\rm top} = 175\,\mathrm{GeV}$. In Fig. 5 we show how, for small values of $\tan\beta$, the ratio h_b/h_τ approaches unity for different input values of $m_{\rm top}$: (a) $\tan\beta = 1.70, 1.92$ with $m_{\rm top} = 175\,\mathrm{GeV}$ (with $\tan\beta = 1.92$, $M_I \simeq 10^{13.05}$ GeV), (b) $\tan\beta = 1.5$ ($M_I \simeq 10^{10.99}\,\mathrm{GeV}$), 1.7 ($M_I \simeq 10^{10.23}\,\mathrm{GeV}$) with $m_{\rm top} = 160\,\mathrm{GeV}$, and (c) $\tan\beta = 1.7$, $m_{\rm top} = 175\,\mathrm{GeV}$, 170 GeV ($M_I \simeq 10^{12.56}\,\mathrm{GeV}$).

The unification of the three Yukawa couplings so far observed at the intermediate scale $M_I = 10^{13.10}\,\mathrm{GeV}$ for the average Tevatron value $m_{\rm top} = 175\,\mathrm{GeV}$, is a consequence of low-energy data and the two Higgs doublets in MSSM. In order to embed such a scenario in SUSY

SO(10), the attractive candidate is the intermediate gauge group G_{224P} having $M_I \simeq 5\times10^{12}$–$2\times10^{14}$ GeV with parity broken at the GUT scale [6]. Similar finding in the case of non-SUSY SO(10) with intermediate symmetry G_{224P} has been reported [11] where the RGEs for the Yukawa couplings in the range $\mu \geq M_I$ and threshold corrections to M_I have also been discussed. The present investigation is a preliminary report in this line in the sense that the two-loop effects in RGEs for Yukawa couplings, the threshold corrections to M_I and the evolution pattern of Yukawa couplings above the scale M_I are yet to be investigated.

For a consistent perturbative treatment of the theory, the top-quark Yukawa coupling must fulfill the condition $h^2_{\rm top}(\mu)/4\pi \leq 1$ at the whole energy range in which the MSSM is considered as a valid effective theory [12, 13]. The requirement of perturbative consistency is naturally fulfilled in the low to intermediate energy regime. Since we are assuming that MSSM is valid up to scale of the order of $M_U \simeq 2\times10^{16}$ GeV, the perturbative consistency condition becomes $h^2_{\rm top}(M_U)/4\pi \leq 1$. Our detailed numerical analysis shows that such triviality bound on the top-quark Yukawa coupling $h_{\rm top}$ determines an upper bound on $m_{\rm top}$ as a function of $\tan\beta$, for values of $M_U \simeq 2\times10^{16}$ GeV. If we wish to have $h_{\rm top}(M_U) \leq 3.54$ for the input values of $\alpha_s(M_Z) = 0.118 \pm 0.004$ and $m_{\rm top} = 175 \pm 6$ GeV, the upper bound on $\tan\beta \geq 1.945^{-0.20}_{+0.40}$. The effect on the variation of m_b has negligible effect on the bound of $h_{\rm top}$ for small $\tan\beta$ values. This scenario contradicts the results of [3]. The inclusion of two-loop contributions does not significantly alter the present one-loop numerical results.

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